Chapter 10

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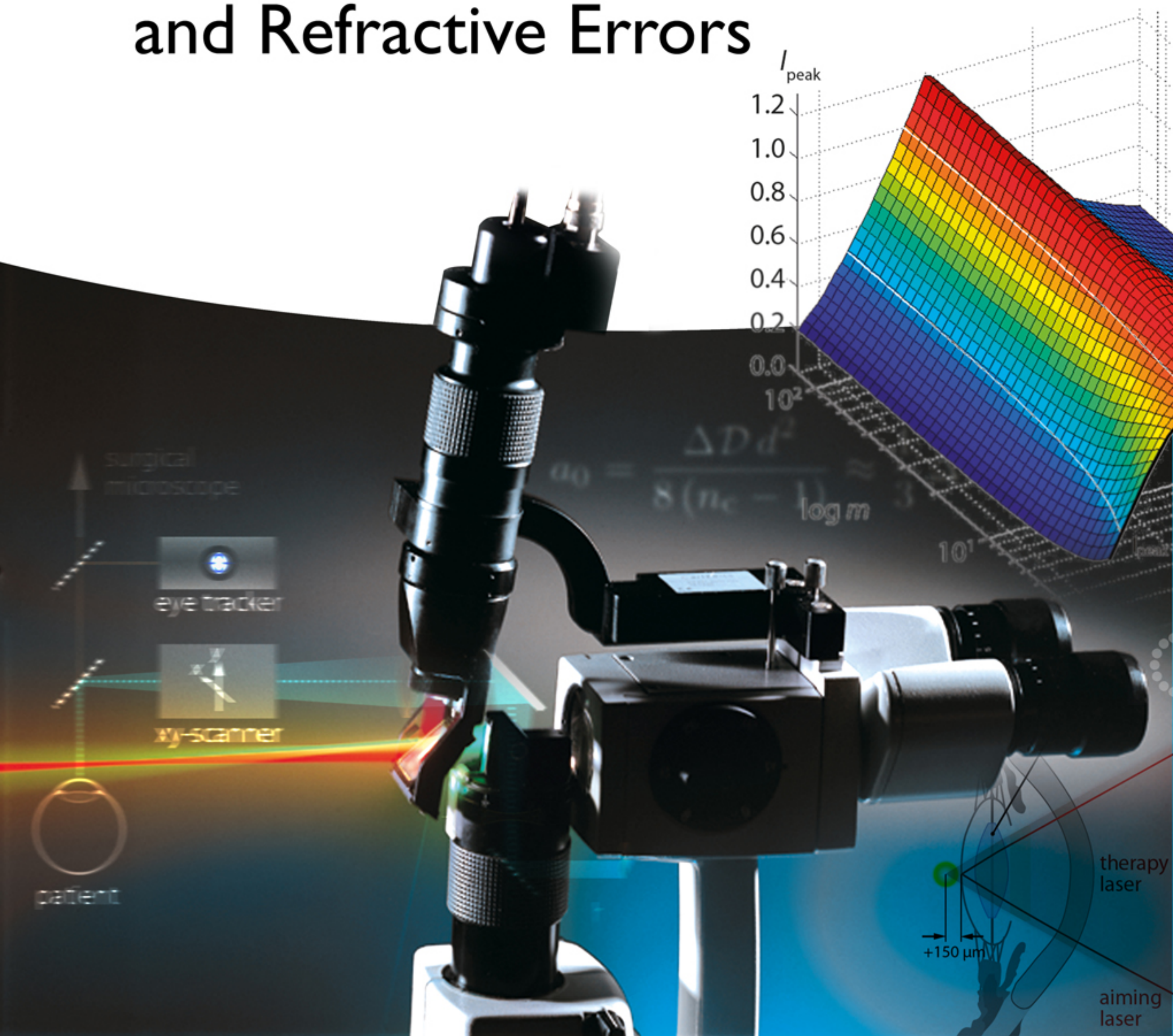
Laser Systems for Treatment of Eye Diseases and Refractive Errors



Solutions to Problems in „Optical Devices in Ophthalmology and Optometry“

Contents

P10.1
Photodynamic therapy I

> Calculate the number of photons deposited into tissue using the parameters in section 10.1.2.1.

Solution:

We use the parameters given in Section 10.1.2.1:

- Exposure: 50 J/cm^2
- Wavelength: 689 nm
- Laser spot diameter: 5 mm.

The number of photons deposited into the tissue can then be calculated via

$$\text{Exposure}\left[\frac{\mathrm{J}}{\mathrm{cm}^2}\right] = \text{Photon flux}\left[\frac{\#}{\mathrm{cm}^2}\right]\cdot\frac{hc}{\lambda}\ , \tag{S10.1}$$

where the photon flux is the number of photons per unit area, and the second term is the energy of a single photon. Therefore, the number of photons deposited is given by

$$\#\text{Photons} = \text{Photonflux}\times\text{Area} = \text{Exposure}\times\frac{\lambda}{hc}\times\text{Area} = 3.4\times10^{19}\ . \tag{S10.2}$$

P10.2
Photodynamic therapy II

> The absolute spectral absorption maximum of the photosensitizer verteporfin is at a wavelength of 400 nm. Hence, an excitation at this wavelength would be very efficient. Why is a wavelength of $\lambda = 689$ nm used instead to destroy choroidal neovascularizations during a photodynamic therapy (PDT)? After the photosensitizer has been injected, we have to wait a while until the laser therapy can be started. Why so?

Solution:

An excitation at $\lambda = 689$ nm is used, as the absorption of most endogenous (tissue) chromophores, such as melanin and xanthophyll, is relatively low in this spectral region (Figure 10.3). As a consequence, the laser light is able to penetrate deeply into the choroid, where choroidal neovascularizations develop. Moreover, at this wavelength, the scattering losses in the anterior part of eye are also lower than in the case of 400 nm (Section 9.2).

The waiting time after the injection of verteporfin (about 5 minutes) is necessary to achieve a maximum concentration difference of the photosensitizer between the newly formed abnormal vessels (neovascularizations) and adjacent tissue (selective accumulation of the photosensitizer in the target region).

P10.3
Photothermal effects

> Estimate the temperature increase in tissue using the parameters in Section 10.2.3.1.

Solution:

We can use the basic law of thermodynamics to determine the temperature rise for the given parameters, which reads

$$\Delta Q = \mathrm{m} \cdot c_{\mathrm{w}} \cdot \Delta T \quad . \tag{S10.3}$$

Here, $m = \rho_{\mathrm{w}} V$ is the tissue mass, ρ_{w} the density, and c_{w} is the specific heat capacity. We use for approximation the water parameters for tissue ($\rho_{\mathrm{w}} \approx 1\ \mathrm{g/cm^3}, c_{\mathrm{w}} = 4.128\ \mathrm{Ws\ g^{-1}\ K^{-1}}$).
The thermal relaxation time of the irradiated retinal tissue can be calculated according to Eq. (9.10) via

$$t_{\mathrm{r}} = \frac{1}{4\,\mu_{\mathrm{a}}^2 \cdot \kappa_{\mathrm{tissue}}} \quad . \tag{S10.4}$$

The absorption coefficient μ_{a} of the main retinal absorption chromophore melanin has a value of $\mu_{\mathrm{a}} \approx 10^3\,\mathrm{cm^{-1}}$ at a wavelength of $\lambda_{\mathrm{L}} = 532\,\mathrm{nm}$ (Figure 9.3). Eye tissue consists mainly of water. Therefore, with regard to the thermal properties, we may approximate the thermal diffusion constant of tissue κ_{tissue} by that of water $\kappa_{\mathrm{water}} = 1.4 \times 10^{-7}\mathrm{m^2/s}$. Equation (S10.4) then yields the thermal relaxation time

$$t_{\mathrm{r}} \approx \frac{1}{4\,\mu_{\mathrm{a}}^2 \cdot \kappa_{\mathrm{water}}} \approx 180\ \mathrm{\mu s} \quad .$$

This value is significantly smaller than the exposure time $t_{\mathrm{exp}} = 100\ \mathrm{ms}$.
Thus, with a given laser spot diameter, we can calculate the volume of the coagulated tissue by means of the thermal penetration depth (Section 9.4.2, Eq. (9.9)) with

$$L_{\mathrm{tpd}} = \sqrt{4 \cdot t_{\mathrm{exp}} \cdot \kappa_{\mathrm{water}}} \approx 240\ \mathrm{\mu m} \quad ,$$

where we used the parameters given in Section 10.2.3.1:

- Exposure time: $t_{\mathrm{exp}} = 100\,\mathrm{ms}$,
- Laser power: $P_{\mathrm{L}} = 150\,\mathrm{mW}$,
- Laser spot diameter: $d_{\mathrm{L}} = 250\ \mathrm{\mu m}$.

If we compare the thermal penetration depth $L_{\mathrm{tpd}} \approx 240$ µm with a spot radius of $d_{\mathrm{L}}/2 = 125\,\mu\mathrm{m}$, it becomes obvious that we have to approximate the coagulated volume by a sphere with radius L_{tpd} so that

$$V = \frac{4\pi}{3} L_{\mathrm{tpd}}^3 \quad .$$

With $\mathrm{m} = \rho_{\mathrm{w}} \cdot V,\ \Delta Q = P_{\mathrm{L}} \cdot t_{\mathrm{exp}}$, and using Eq. (S10.3), we obtain a temperature increase of $\Delta T = 62\,\mathrm{K}$.

P10.4
Retinal photocoagulation I

Compare for typical parameters of standard and short-pulse photocoagulation the temperature increase, the diameter of coagulated tissue, and the deployed energy.

Solution:

Standard mode photocoagulation (single pulse)
The parameters for this mode are identical with the ones used in Problem P10.3, so we have

- Exposure time: $t_{\mathrm{exp}} = 100\,\mathrm{ms}$,
- Laser power: $P_{\mathrm{L}} = 150\,\mathrm{mW}$,
- Laser spot diameter: $d_{\mathrm{L}} = 250\,\mu\mathrm{m}$,
- Thermal penetration depth: $L_{\mathrm{tpd}} = \sqrt{4 \cdot t_{\mathrm{exp}} \cdot \kappa_{\mathrm{water}}} \approx 240\ \mu\mathrm{m}$.

If we compare the thermal penetration depth with the laser spot radius (125µm), we see that the coagulated volume has to be approximated by a sphere with radius L_{tpd} so that

$$V = \frac{4\pi}{3} L_{\mathrm{tpd}}^3 = 5.8 \times 10^{-5}\mathrm{cm}^3 \quad .$$

With $\mathrm{m} = \rho_{\mathrm{w}} \cdot V,\ \Delta Q = P_{\mathrm{L}} \cdot t_{\mathrm{exp}}$, and using Eq. (S10.3), we obtain a temperature increase of ΔT = 62K. The total developed energy is 15 mJ. The diameter of the coagulated tissue is about 2 $L_{\mathrm{tpd}} \approx 500\,\mu\mathrm{m}$.

Short-pulse mode photocoagulation (micro-pulses in a pulse train)
The parameters for this mode (micro-pulses in a pulse train) are according to Section 10.2.3.2:

Pulse train:

- Duration: 0.2 s ,
- Repetition frequency: 500 Hz (i.e., 50 single pulses/train)

Single micro-pulse:

- Exposure time: $t_{\mathrm{exp}} = 0.2\,\mathrm{ms}$ (is equal to the pulse duration) ,
- Laser power (single micro-pulse): $P_{\mathrm{L}} = 1500\,\mathrm{mW}$,
- Laser spot diameter: $d_{\mathrm{L}} = 250\,\mathrm{\mu m}$.

With $t_{\mathrm{exp}} = 0.2\,\mathrm{ms}$, we obtain for the thermal penetration depth in the short-pulse mode

$$L_{\mathrm{tpd}} = \sqrt{4 \cdot t_{\mathrm{exp}} \cdot \kappa_{\mathrm{water}}} \approx 11\ \mathrm{\mu m} \ .$$

If we compare this thermal penetration depth with the laser spot radius (125 µm), we see that the coagulated volume must be approximated by a cylinder with radius $d_{\mathrm{L}}/2$ and height L_{tpd} so that

$$V = \frac{\pi}{4} d_{\mathrm{L}}^2 \cdot L_{\mathrm{tpd}} \approx 5.4 \times 10^{-7}\mathrm{cm}^3 \ .$$

Thus, in a short-pulse mode photocoagulation, a much smaller volume is thermally influenced.

With $\mathrm{m} = \rho_{\mathrm{w}} V$, $\Delta Q = P_{\mathrm{L}}\, t_{\mathrm{exp}}$, and using Eq. (S10.3), we obtain a temperature increase of $\Delta T = 133\,\mathrm{K}$ per single micro-pulse. The total deployed energy per single pulse is 0.3 mJ. The total deployed energy is 15 mJ.

The diameter of coagulated tissue perpendicular to the laser beam direction is approximately $d_{\mathrm{L}} = 250\,\mathrm{\mu m}$. The corresponding extension parallel to the laser beam direction (cylinder axis of coagulated volume) is approximately given by $L_{\mathrm{tpd}} = \sqrt{4\, t_{\mathrm{exp}}\, \kappa_{\mathrm{water}}} \approx 11\ \mathrm{\mu m}$.

P10.5
Retinal photocoagulation II

> For photocoagulation on the retina, an indirect ophthalmoscope can be used. With a 20 D ophthalmoscopy lens, a real (aerial) image of the retina is obtained. What is the size of the laser spot on the retina for an emmetropic eye if the laser beam diameter in the plane of the aerial image is 1 mm ?

Solution:

The laser spot size (w_{f}) on the retina of an emmetropic eye using an indirect ophthalmoscope (Figure 10.9) can be calculated from the spot size (w_{i}) at the plane of the aerial image of the fundus using Eq. (10.7). We have

$$w_{\mathrm{f}} = \frac{w_{\mathrm{i}}}{\beta} = -w_{\mathrm{i}} \cdot \frac{\mathcal{D}_{\mathrm{oph}}}{\mathcal{D}_{\mathrm{eye}}} \ . \tag{S10.4}$$

With $2w_{\mathrm{i}} = 1\,\mathrm{mm}$, $\mathcal{D}_{\mathrm{oph}} = 20\,\mathrm{D}$, and $\mathcal{D}_{\mathrm{eye}} = 60$ D, we obtain for the spot diameter on the retina $2w_{\mathrm{f}} = 330\,\mathrm{\mu m}$.

P10.6
Step-index fiber

1. Consider a step-index fiber with a numerical aperture of $\mathrm{NA} = 0.2$ and a refractive index of the fiber core of $n_\mathrm{c} = 1.5$. What is the refractive index of the cladding?
2. An expanded and collimated laser beam with a waist radius of $w_0 = 0.5\,\mathrm{cm}$ and a beam divergence of $\varepsilon = 0.2\,\mathrm{mrad}$ shall be coupled into an optical fiber with $\mathrm{NA} = 0.2$ and a core diameter of $d_\mathrm{c} = 50\,\mu\mathrm{m}$. What are the minimum and maximum focal lengths of the fiber coupling lens to achieve a loss-free coupling-in? Why is it more advantageous for the instrument design to use lenses with a shorter focal length? Coupling losses due to Fresnel reflection at the surface shall be considered here as second-order. Is that justified?

Solution:

1. Using Eq. (10.4)

$$\mathrm{NA} = n_0 \cdot \sin\alpha_\mathrm{max} = \sqrt{n_\mathrm{c}^2 - n_\mathrm{cl}^2} \ , \tag{S10.5}$$

we find for the refractive index of the cladding

$$n_\mathrm{cl} = \sqrt{n_\mathrm{c}^2 - \mathrm{NA}^2} = \sqrt{1.5^2 - 0.2^2} = 1.48 \ . \tag{S10.6}$$

The difference in refractive indices between cladding and core is thus relatively low (1.5%).

2. For an efficient coupling of the laser beam into the fiber, conditions (10.5) and (10.6) should be fulfilled. Determining the minimum focal length from Eq. (10.5), we obtain $\theta = \arctan\,(w_\mathrm{L}/f) < \alpha_\mathrm{max}$, where $\alpha_\mathrm{max} = \arcsin\,(\mathrm{NA}) = 11.5°$. To give some safety cushion, we use $\theta = 10°$ and calculate the focal length of the lens to be used as

$$f = \frac{w_\mathrm{L}}{\tan\theta} = \frac{0.5}{0.176}\ \mathrm{cm} \approx 3\ \mathrm{cm} \ .$$

Checking the fulfillment of condition (10.6), we find

$$w_0 = f\varepsilon = 6\ \mu\mathrm{m} \le 0.35\ d_\mathrm{c} = 17\ \mu\mathrm{m} \ .$$

We see that the condition is satisfied. We then determine the maximum of the focal length using Eq. (10.6) as

$$w_0 = 0.35\ d_\mathrm{c} = 17\ \mu\mathrm{m} \ .$$

The maximum focal length could be

$$f = \frac{w_0}{\varepsilon} = 8.5\ \mathrm{cm} \ .$$

Condition (10.5) is also satisfied when using this focal length, since

$$\theta = \arctan\left(\frac{w_{\mathrm{L}}}{f}\right) = 3.6^\circ < \alpha_{\max} = 11.5^\circ \quad .$$

We have thus the choice of either using a longer (8.5 cm) or shorter focal length lens (3 cm). A shorter focal length lens is however preferable, as it focuses the laser beam to a smaller spot which allows large tolerance in the alignment of the optical fiber in the transverse direction (xy) at the focal plane (condition (10.6)). Coupling losses due to Fresnel reflection can indeed be considered as second order, because these losses happen at both end surfaces of the optical fiber with approximately a few percent for uncoated fiber end surfaces (almost normal incidence, see Eq. (A65)). Due to the short length of the optical fiber, other losses in the fiber can also be neglected.

P10.7
Laser link

> Calculate a suitable layout for the laser link shown in Figure 10.7 using a diode laser and as parameters the data given in Section 10.2.3.1. Assume an optical fiber diameter of 100 μm.

Solution:

The essential parameters for a laser link system are

- a fiber core diameter of the optical fiber: $d_{\mathrm{F}} = 100$ μm,
- a distance link mirror → slit lamp focal plane: $l_{\mathrm{M-SL}} = 50\,\mathrm{mm}$ (realistic assumption),
- a laser contact lens with magnification factor of laser spot on retina: $\beta = 1\times$ (realistic assumption),
- an emmetropic eye with power $\mathcal{D}_{\mathrm{eye}} = 60\,\mathrm{D}$, and
- a numerical fiber aperture of $\mathrm{NA} = 0.1$.

We want to determine the principle optical layout of the laser link (zoom) system according to Figure 10.7 (Section 10.2.3.1) by means of the ABCD matrix method under the condition that we obtain a variable spot size (diameter) on the retina (i.e., in the slit lamp focal plane) d_{retina} of 50 to 500 μm. The optical design problem can also be described as how to obtain a sharp image of the fiber exit surface in the retinal "plane" (in the slit lamp focal plane) with a variable magnification of $0.5\times$ to $5\times$ (Figure 10.7).
According to Section 6.2.3.4, the easiest approach is to collimate the radiation exiting the fiber, change the diameter by a symmetrical afocal zoom system and then refocus the collimated beam into the focal plane. Figure S10.1 (not to scale) shows the basic layout. For this approach, the free working distance of the focusing lens must be

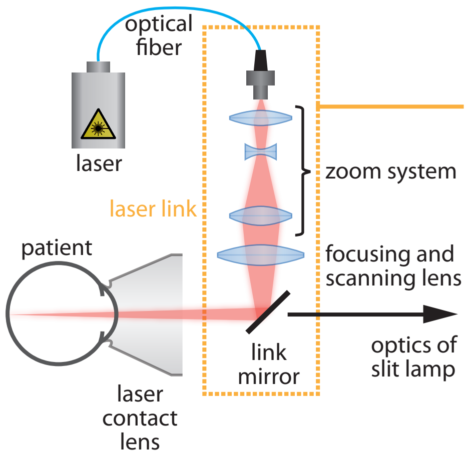


Figure 10.7 Schematic setup of a laser link

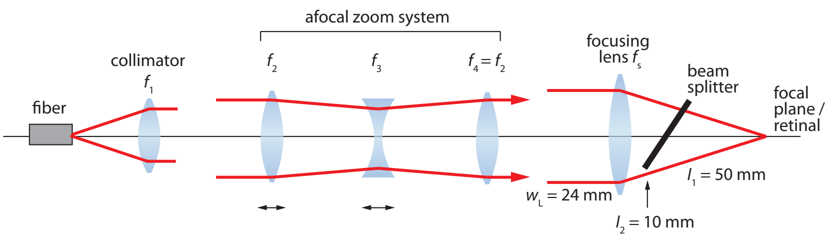


Figure S10.1 Collimation of light emitted from a fiber.

larger than $l_1 = 50\,\text{mm}$. If an additional distance of $l_2 = 10\,\text{mm}$ is assumed to take the beam splitter geometry into account, the lens has a focal length of $f_5 = 60\,\text{mm}$. This produces a beam diameter of $w_\text{L} = 24\,\text{mm}$ at the lens.

The symmetric zoom must be configured with a zooming factor of $\text{M} = 10$. Corresponding to the symmetry, the 3-lens system must have a magnification of $\Gamma = 1$ for an incoming beam diameter of

$$w_\text{in} = \frac{w_\text{L}}{\sqrt{M}} = 7.6\ \text{mm} \quad .$$

If a typical numerical aperture of the fiber of $\text{NA} = 0.1$ is assumed, we find the necessary focal length of the collimator to be

$$f_1 = \frac{w_\text{in}}{2\,\text{NA}} = 37.9\ \text{mm} \quad .$$

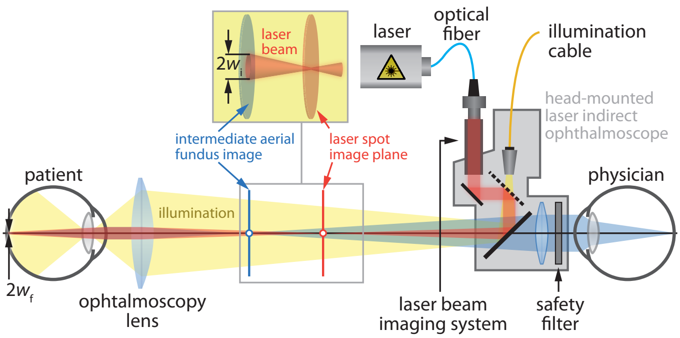


Figure 10.9 Schematic setup of a head-mounted indirect ophthalmoscope (LIO)

With Eqs. (6.42) and (6.43), we can finally determine the local lengths of the zoom system for an assumed overall length of $L = 100\,\text{mm}$ as $f_2 = f_4 = 96.2$ mm and $f_3 = -23.1\,\text{mm}$.

For these numbers, the length of the system from the fiber to the beam splitter is 138 mm. In principle, the system can be designed easier if lenses 4 and 5 are merged. But this reduction of components leads to a system which is more complicated to adjust. If lens 4 is moved, lenses 1 and 2 can be combined and the layout looks more or less like the one in Figure 10.7.

P10.8
Head-mounted laser indirect ophthalmoscope (LIO)

1. Calculate the laser focus diameter on the retina of the head-mounted laser indirect ophthalmoscope shown in Figure 10.9. Assume a standard ophthalmoscopy lens (20 D) to be used and an optical fiber diameter of 100 µm. Design the laser beam imaging system so that a laser spot image of 1 mm is obtained in the (intermediate) aerial image plane of the fundus of an emmetropic eye. Consider typical distances (arm lengths etc.).
2. What is the pointing stability of the laser spot on the retina if the head of the ophthalmologist moves by 1°. What happens if the ophthalmoscopy lens is tilted by 5° during treatment?

Solution:

1. The essential parameters for a head-mounted laser indirect ophthalmoscope shown in Figure 10.9 are:
   - Fiber core diameter of the optical fiber $(\text{NA} = 0.1) : d_f = 100\,\mu\text{m}$.
   - We assume a comfortable viewing distance of $s_{nv} = 250\,\text{mm}$.

- Magnification (laser spot on retina vs. laser spot in intermediate aerial image) for an ophthalmoscopy lens of 20 D: $\beta = -0.33\times$.

We assume that a single lens with focal length f_{L} placed in front of the fiber can bring the laser spot image plane onto the intermediate aerial fundus image plane (red arrow in Figure 10.9). If the conditions for such an imaging of the fiber and surface with
1. the magnification $\beta_{\mathrm{L}} = \frac{s'}{s} = -10$ as discussed in Eq. (A15) and
2. the imaging equation $\frac{1}{s'} - \frac{1}{s} = \frac{1}{f_{\mathrm{L}}}$ as discussed in Eq. (A14)
are combined (eliminate s), we get for the focal length

$$f_{\mathrm{L}} = \frac{s'}{1 - \beta_{\mathrm{L}}} \quad .$$

If we assume, as stated above, a comfortable viewing distance of $s' = s_{\mathrm{nv}} = 250\,\mathrm{mm}$, we get

$$f_{\mathrm{L}} = \frac{250\ \mathrm{mm}}{11} \approx 23\ \mathrm{mm} \quad .$$

Therefore, the system can be built with a single lens which has a focal length of 23 mm.
The fiber end plane has to be placed at a distance $s = -25$ mm in front of the principal plane of the lens. Then, in the laser spot image plane (approximately the aerial image plane of the fundus of an emmetropic eye at a comfortable viewing distance of $s_{\mathrm{nv}} = 250$ mm), a laser spot image of 1 mm is obtained.

2. A tilt of the ophthalmologist's head by $1°$ results in a shift of the laser spot in the intermediate plane by

$$\Delta x_{\mathrm{i}} = \tan 1^{\circ} \cdot 250\ \mathrm{mm} = 4.36\ \mathrm{mm} \quad .$$

The shift of the laser spot will be de-magnified by the ophthalmoscopy lens so that

$$\Delta x_{\mathrm{retina}} = \Delta w_{\mathrm{i}} \cdot (-0.33) \approx -1.4\ \mathrm{mm} \quad .$$

Hence, a tilt by the head of the ophthalmologist by $1°$ results in a laser spot shift on the retina of an emmetropic eye by almost 1.4 mm.

A tilt of the 20 D ophthalmoscopy lens ($f_{\mathrm{oph}} = \mathcal{D}_{\mathrm{oph}}^{-1} = 50\ \mathrm{mm}$) by $5°$ is equivalent to a laser spot shift in the intermediate image plane of

$$\Delta x_{\mathrm{i}} = \tan 5^{\circ} \cdot 50\ \mathrm{mm} = 4.73\ \mathrm{mm} \quad .$$

Again, we have a de-magnification effect on the retina because of the magnification factor $\beta = -0.33\times$ of the ophthalmoscopy lens which leads to

$$\Delta x_{\mathrm{retina}} = \Delta x_{\mathrm{i}} \cdot (-0.33) \approx 1.56\ \mathrm{mm}.$$

A tilt of the ophthalmoscopy lens by $5°$ thus results in a laser spot shift on the retina of an emmetropic eye by about 1.6 mm. Any distortions due to coma and other aberrations are not included in this calculation.
These estimates show that the use of an indirect ophthalmoscope requires quite some skill and a "steady hand".

P10.9
Photoablation (blow-off model)

Calculate the ablation rate per laser pulse L_{abl} in corneal stroma according to the blow-off model for an ArF excimer laser ($\lambda = 193$ nm) and a KrF excimer laser ($\lambda = 248$ nm). The absorption coefficients of stroma are $\mu_{\mathrm{a}}(193\,\mathrm{nm}) = 29,000$/cm and $\mu_{\mathrm{a}}(248\,\mathrm{nm}) = 290$/cm. The corresponding exposure thresholds are $\Phi_{\mathrm{th}}(193\,\mathrm{nm}) = 50\,\mathrm{mJ/cm}^2$ and $\Phi_{\mathrm{th}}(248\,\mathrm{nm}) = 500\,\mathrm{mJ/cm}^2$. In both cases, the laser exposure is assumed to be $\Phi = 600\,\mathrm{mJ/cm}^2$.

Solution:

The ablation depth for a given laser exposure Φ can be calculated using Eq. (9.12), that is,

$$L_{\mathrm{abl}} = \frac{1}{\mu_{\mathrm{a}}(\lambda)} \ln\left(\frac{\Phi}{\Phi_{\mathrm{th}}}\right) \quad ,$$

where Φ_{th} is the threshold exposure required to achieve ablative material removal. Calculating the ablation depth for each wavelength using the given parameters leads to

- $L_{\mathrm{abl}}(193\ \mathrm{nm}) \approx 0.9\ \mu\mathrm{m}$,
- $L_{\mathrm{abl}}(248\ \mathrm{nm}) \approx 6.3\ \mu\mathrm{m}$.

P10.10
Photoablation (thermal effects)

An ArF excimer laser has a typical pulse length of 20 ns. Why are thermal effects of this laser negligible when it is used for photoablation of corneal stroma (thermal diffusion constant of stroma $\kappa = 1.5 \times 10^{-7}\,\mathrm{m}^2$/s; absorption coefficient $\mu_{\mathrm{a}}(193\,\mathrm{nm}) = 29,000$/cm)?

Solution:

The thermal effect of the laser can be described by the thermal penetration depth according to Eq. (9.9) via

$$L_{\mathrm{tpd}} = \sqrt{4\kappa t_{\mathrm{exp}}} \quad .$$

With $\kappa = 1.5 \times 10^{-7} \mathrm{m}^2/\mathrm{s}$ and $t_{\mathrm{exp}} = 20\,\mathrm{ns}$, we obtain $L_{\mathrm{tpd}} = 0.05\,\mu\mathrm{m}$. Moreover, the optical penetration depth is calculated using Eq. (9.3) as

$$\delta_{\mathrm{a}} = \frac{1}{\mu_{\mathrm{a}}(\lambda)} \quad .$$

With $\mu_{\mathrm{a}}(193\mathrm{nm}) = 2900\ \mathrm{cm}^{-1}$, we calculate the optical penetration depth to $\delta_{\mathrm{a}} = 0.35\ \mu\mathrm{m}$. Therefore, in comparison to the optical penetration depth, it is safe to neglect thermal effects due to a very short thermal penetration depth.

P10.11
Photoablation (refractive change)

> In refractive corneal surgery, an excimer laser is used to increase the central radius of curvature of the corneal front surface by $+3\ \%$.
> 1. Let us assume that the treated eye can be described by the Exact Gullstrand Eye model. Calculate the change of the refractive power of the corneal front surface.
> 2. For what kind of refractive error is such a treatment useful?

Solution:

1. Referring to the solution of Problem (PI.4), a relative change of the central radius of curvature of the cornea r_c of a Gullstrand Eye model (Table 2.1) by +1% results in a change of refractive power by $\Delta\mathcal{D}_c = -0.46\,\mathrm{D}$. Therefore, an increase of the radius by 3% amounts to $\Delta\mathcal{D}_c \approx -1.4\,\mathrm{D}$.

2. Since the refractive power of the eye is reduced, such a treatment would be useful for a myopic correction.

P10.12
Photoablation (required precision)

> Let us consider a laser system which allows photoablation with a precision of $\pm 5\,\mu\mathrm{m}$.
> 1. What is the maximum precision during refractive surgery which can be achieved with this instrument for a typical ablation zone diameter of $6\,\mathrm{mm}$?
> 2. Does it make sense to have an ablation precision of $\pm 1\,\mu\mathrm{m}$?

Solution:

1. According to the Munnerlyn equation (10.9), the refractive power change is given by
$$\Delta\mathcal{D} = \frac{3a_0}{d^2} \ .$$
For an ablation precision of $a_0 = \pm 5\ \ \mu\mathrm{m}$ and $d = 6\,\mathrm{mm}$, we obtain for the maximum achievable precision of refractive power change $\Delta\mathcal{D}\ \ \pm 0.42\,\mathrm{D}$. This value is above the noticeable refractive error threshold of ± 0.25 D. Therefore, a precision of ± 5 µm of the ablation laser system is not enough.

2. For an ablation precision of $a_0 = \pm 1\ \mu\mathrm{m}$, the change of the refractive power is $\Delta\mathcal{D} = \pm 0.08\,\mathrm{D}$. This value is well below the noticeable refractive error threshold of $\pm 0.25\,\mathrm{D}$, and thus a precision of $\pm 1\,\mu\mathrm{m}$ for an ablation laser makes perfect sense.

P10.13
Photoablation (LASIK I)

A myopic eye has a central corneal thickness of 530 µm and shall be treated with LASIK. The flap thickness is 160 µm and the diameter of the optical zone 6 mm. Calculate the maximum possible correction of the refractive power under consideration of the stability limit of the corneal thickness.

Solution:

The maximum achievable correction of refractive power considering the stability of the cornea after surgery can be calculated using Eq. (10.23), that is,

$$\Delta\mathcal{D} = \frac{3a_{0,\max}}{d^2} \quad ,$$

with $a_{0,\max} = L_{\mathrm{cc}} - L_{\mathrm{flap}} - L_{\mathrm{rs}} = 530\ \mu\mathrm{m} - 160\ \mu\mathrm{m} - 250\ \mu\mathrm{m} = 120\ \mu\mathrm{m}$. Using the given parameters d and $a_{0,\max}$, we obtain for the maximum achievable refractive correction $\Delta\mathcal{D}$ = 10 D.

P10.14
Photoablation (LASIK II)

A myopic patient desires a LASIK refraction correction. The required refractive power of the corrective lens is determined preoperatively by subjective refraction using a phoropter. The trial lenses of the phoropter are placed in front of the eye at a distance of $L = 12$ mm from the corneal vertex. The patient achieves the best far distance visual acuity with a trial lens of back vertex power $\mathcal{D}'_{\mathrm{v}} = -6$ D. What is the necessary ablation depth in the center if the physician chooses the Mullerlyn ablation profile with an optical zone diameter of 6 mm for myopia correction?

Solution:

Since the surgery is performed at the plane of the corneal vertex where the refractive correction happens, it is at first necessary to change the measured value of -6 D obtained at a distance $L_{\mathrm{c1}} = 12$ mm to the new distance, that is, $L_{\mathrm{c2}} = 0$. Using Eq. (5.36)

$$\mathcal{D}'_{\mathrm{v}}(L_{\mathrm{c2}}) = \frac{\mathcal{D}'_{\mathrm{v}}(L_{\mathrm{c1}})}{1 + \mathcal{D}'_{\mathrm{v}}(L_{\mathrm{c1}}) \cdot (L_{\mathrm{c2}} - L_{\mathrm{c1}})}$$

with $L_{\mathrm{c1}} = 12$ mm, $L_{\mathrm{c2}} = 0$, and $\mathcal{D}'_{\mathrm{v}}(L_{\mathrm{c1}}) = -6$ D, we obtain $\mathcal{D}'_{\mathrm{v}}(L_{\mathrm{c2}}) = -5.6$ D. Then, the necessary ablation depth at the corneal center can in turn be found by using

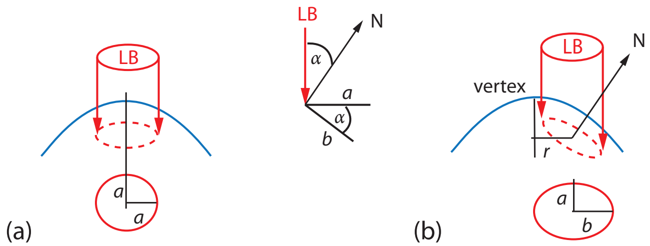


Figure S10.2 Geometery of a laser beam which is incident to the corneal surface. (a) Incidence is central of the surface. (b) Incidence is shifted from the corneal vertex. The illuminated corneal surface is thus ellipsoidal.

the Munnerlyn formula (10.9). We thus have

$$a_0 = \frac{1}{3}\Delta\mathcal{D}\cdot d^2 = -\frac{1}{3}\cdot 5.6\ \mathrm{m}^{-1}(6\times 10^{-3}\ \mathrm{m})^2 \approx 67\ \mu\mathrm{m}\ .$$

P10.15
Photoablation (exposure correction)

> The photoablation rate per laser pulse depends on the applied exposure (see Eq. (9.12)). As the surface of the cornea is curved, the beam waist diameter changes as a function of the location on the corneal surface. Without correction, the ablation rate would gradually decrease from the corneal center to the periphery, as the effective exposure drops. Calculate the necessary correction factor for the laser energy as a function of the laser spot decentration from the corneal vertex.

Solution:

When a laser beam with radius a is incident on the center of the corneal vertex, it forms a circular spot of radius a (Figure S10.2a). When the laser beam is incident at a de-centered distance r from the corneal vertex, it forms an elliptical laser spot with two half-axes a and b. This is because the laser beam is incident at an angle $\alpha(r)$ which is different from perpendicular to the surface of the cornea, which then increases the area and thus decreases the effective exposure. The angle $\alpha(r)$ can be calculated approximately as a function of the decenter distance as $\alpha(r) = r/R$, where R is the radius of curvature of the cornea. From the angular eccentricity of the ellipse, we get $\cos\alpha = a/b$. The reduction in exposure efficiency due to non-normal incidence can be determined by the ratio of the exposure at the two distances as

$$\beta(r) = \frac{\Phi(r)}{\Phi(0)} = \frac{\pi a^2}{\pi ab} = \frac{a}{b} = \cos\left(\frac{r}{R}\right)\ . \tag{S10.7}$$

Therefore, a correction factor of $1/\beta(r)$ is necessary to obtain a constant ablation rate across the corneal surface.

P10.16
Photoablation (OPD) and wavefront aberrations

> The aberrations of the eye can be described, e.g., by means of the optical path difference OPD (section 5.3.1.3). From the OPD, we immediately obtain the data for the necessary ablation profile in a wavefront-guided refractive corneal surgery. The OPD map shows directly at which locations of the cornea how much stromal material needs to be removed in order to achieve a reduction of the optical path length and thus for the correction of the aberrations. Verify the "rule of 3" which states that for the correction of $+1\,\mu\mathrm{m}$ OPD or ($-1\,\mu\mathrm{m}$ wavefront aberration) approximately $3\,\mu\mathrm{m}$ corneal stroma (refractive index $n_\mathrm{c} = 1.376$) must be removed.

Solution:

According to Eq. (5.12), the optical path difference is given by

$$\mathrm{OPD} = \mathrm{OPL}_1 - \mathrm{OPL}_2 \ ,$$

with OPL_2 being the optical path length in the cornea after the ablation of a surface layer of thickness d and OPL_1 being the optical path length in the cornea with thickness d_c before ablation.

We thus obtain

$$\mathrm{OPD} = \mathrm{OPL}_2 - \mathrm{OPL}_1 = d_\mathrm{c} n_\mathrm{c} - [d + (d_\mathrm{c} - d) n_\mathrm{c}] = -d \cdot (n_\mathrm{c} - 1)$$
$$\Rightarrow \ d = \frac{\mathrm{OPD}}{n_\mathrm{c} - 1} = \frac{1\ \mu\mathrm{m}}{0.376} = 2.66\ \mu\mathrm{m} \approx 3\ \mu\mathrm{m} \ .$$

For a correction of $\pm 1\,\mu\mathrm{m}$ OPD, approximately $3\,\mu\mathrm{m}$ corneal stroma must be removed.

P10.17
Super-Gaussian beams

> Assume a Gaussian and a super-Gaussian beam ($m = 20$, $w_0 = 1\,\mathrm{mm}$) with equal energy, that is:
> $$\int_{-\infty}^{\infty} I_{m=2}(r)\,\mathrm{d}r = \int_{-\infty}^{\infty} I_{m=20}(r)\,\mathrm{d}r \ . \qquad \text{(P10.1)}$$
> Calculate the peak intensity for both beam profiles in the focus of a lens ($f = 10\,\mathrm{cm}$) with a pupil radius of $1\,\mathrm{mm}$.

Solution:

Let us evaluate the power (energy) of each beam by integrating the intensity profile over the focal plane.

For a Gaussian beam, we get

$$\begin{aligned} P_{\mathrm{G}} &= \int_0^\infty I_{m=2}(r)\, 2\pi r\, \mathrm{d}r = \int_0^\infty I_{\mathrm{G}} \cdot \exp\left(-2\left(\frac{r}{w_0}\right)^2\right) 2\pi r\, \mathrm{d}r \\ &= \pi w_0^2 I_{\mathrm{G}} \quad . \end{aligned} \tag{S10.8}$$

For the super-Gaussian beam, it follows that

$$\begin{aligned} P_{\mathrm{SG}} &= \int_0^\infty I_{m=20}(r)\, 2\pi r\, \mathrm{d}r = \int_0^\infty I_{\mathrm{SG}} \cdot \exp\left(-2\left(\frac{r}{w_0}\right)^m\right) 2\pi r\, \mathrm{d}r \\ &= \pi w_0^2\, I_{\mathrm{SG}} \cdot \frac{\Gamma(1/m)}{m \cdot 2^{1/m}} \quad , \end{aligned} \tag{S10.9}$$

where Γ is the Gamma function defined as

$$\Gamma(z) = \int_0^\infty t^{z-1} \cdot \mathrm{e}^{-t} \mathrm{d}t \quad .$$

Equations (S10.8) and (S10.9) lead to

$$\frac{P_{\mathrm{SG}}}{P_{\mathrm{G}}} = \frac{m \cdot 2^{1/m}}{\Gamma(1/m)} = 1.06 \tag{S10.10}$$

for $m = 20$.

P10.18
YAG Laser Capsulotomy

> Calculate for typical values of f_{obj} and f_{cl} ("cl" means capsulotomy lens) the spot size for a YAG laser beam assuming that the aperture of the objective lens is fully used. How long must the focus shift be to cover $2/3$ of the diameter of the backside capsule (same for front side capsule). Use the parameters from Table 10.9.

Solution:

In YAG laser capsulotomy, an Abraham capsulotomy lens with a refractive power of 66 D and laser spot magnification of $0.56\times$ are normally used. With the parameters

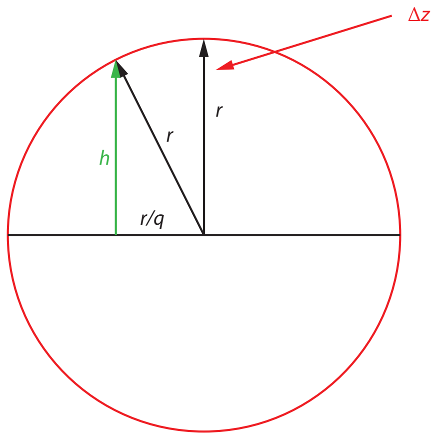


Figure S10.3 Geometric conditions for Problem P10.18

from Table 10.9 for the spot size in air ($8\ \mu m$), the spot size after focusing would be $0.56 \cdot 8\ \mu m = 4.5\ \mu m$.

In order to calculate the necessary focus shift for the laser to cover the depth range due to the curvature of the back (front) side capsule, we use Figure S10.3. According to the geometry, we have the following relationship ($q = 3$)

$$h = r \cdot \sqrt{1 - (1/q^2)} = r \cdot 0.943 \ .$$

From this, we can easily calculate the focus shift

$$\Delta z = r - h = r \cdot 0.0572 \ .$$

We use the radii of the Simplified Gullstrand Eye #2 for relaxed vision (Table 2.2), that is,

- Radius of back surface of eye lens: $r = -6\,\text{mm}$,
- Radius of front surface of eye: $r = 10\,\text{mm}$,

and obtain for the focus shift

$$\Delta z(\text{backside}) = 340\ \mu m \ ,$$
$$\Delta z(\text{frontside}) = 572\ \mu m \ .$$

Thus, the optical setup needs to allow for a focus shift of up to $600\ \mu m$.

P10.19
Laser-induced plasma

Calculate the maximum expansion Δz_{max} of the laser-induced plasma for a laser pulse with constant energy with the following parameter set:
- Pulse length $\tau = 200\,\mathrm{fs}$, $500\,\mathrm{fs}$, and $1000\,\mathrm{fs}$
- Wavelength $\lambda = 1\,\mu\mathrm{m}$
- Beam waist $w_0 = 1\mu\mathrm{m}$

Assume that for $\tau = 1000\,\mathrm{fs}$ we have $I_{\mathrm{peak}} = I_{\mathrm{th}}$. Calculate also the total energy deposition.

Solution:

The maximum plasma expansion can be calculated via Eq. (9.14) which reads

$$\Delta z_{\mathrm{max}} = z_{\mathrm{R}} \cdot \sqrt{\frac{I_{\mathrm{peak}}}{I_{\mathrm{th}}} - 1} \ . \tag{S10.11}$$

We can use the relation $I_{\mathrm{peak}} = I_{\mathrm{th}}$ at $\tau = 1000\,\mathrm{fs}$ as a reference to derive the relationship between the two quantities at different pulse lengths. For lasers in the ps-range, the threshold exposure for an optical breakdown depends on the pulse duration according to (Eq. (9.13))

$$\Phi_{\mathrm{th}} \propto \sqrt{\tau} \ .$$

Using the energy-time relationship, the dependency of the threshold intensity would be

$$I_{\mathrm{th}} \propto \frac{1}{\sqrt{\tau}} \ .$$

Comparing the threshold intensities of different pulse durations using the above relation gives

$$\frac{I_{\mathrm{th},2}}{I_{\mathrm{th},1}} = \sqrt{\frac{\tau_1}{\tau_2}} \ . \tag{S10.12}$$

Moreover, we assume the pulses to have constant energy. For a Gaussian pulse, the energy can be calculated as

$$E_{\mathrm{p}} \propto \tau \cdot I_{\mathrm{peak}} \ .$$

By the same token, comparing the peak intensities of different pulse durations using the above relation gives

$$\frac{I_{\mathrm{peak},2}}{I_{\mathrm{peak},1}} = \frac{\tau_1}{\tau_2} \ . \tag{S10.13}$$

From Eqs. (S10.12) and (S10.13), we obtain

$$\frac{I_{\mathrm{peak},2}}{I_{\mathrm{th},2}} = \frac{I_{\mathrm{peak},1}}{I_{\mathrm{th},1}} \cdot \sqrt{\frac{\tau_1}{\tau_2}} \ . \tag{S10.14}$$

From the reference $I_{\text{peak},1} = I_{\text{th},1}$ at $\tau_1 = 1000\,\text{fs}$, Eq. (S10.14) reduces to

$$\frac{I_{\text{peak},2}}{I_{\text{th},2}} = \sqrt{\frac{\tau_1}{\tau_2}} \quad . \tag{S10.15}$$

Using all the parameters given, the Rayleigh length can be calculated (Eq. (9.15)) via

$$z_{\text{R}} = \frac{\pi w_0^2}{\lambda} = 3.14\ \mu\text{m} \quad .$$

Substituting Eq. (S10.15) and z_{R} in Eq. (S10.11), the maximum plasma expansions for the pulse lengths τ result in

$$\Delta z_{\text{max}}\ (\tau = 200\ \text{fs}) = 3.49\ \mu\text{m} \quad ,$$
$$\Delta z_{\text{max}}\ (\tau = 500\ \text{fs}) = 2.02\ \mu\text{m} \quad .$$

For $\tau = 1000\,\text{fs}$, where $I_{\text{peak}} = I_{\text{th}}$, we obtain $\Delta z_{\text{max}} = 0$, which would be unreasonable. According to Docchio F. *et al* [1)] , the uncertainty in the length of the plasma for near-threshold intensity is very high. However, using the intrinsically statistical nature of the breakdown process, it is possible to say there is some probability that the plasma expands to an axial distance of 0.5 $z_{\text{R}} = 1.6\,\mu\text{m}$.

To calculate the total energy deposition, we use the threshold exposure for the fs range according to Table 9.1, which is $\Phi_{\text{th}} = 2\text{J/cm}^2$. With a spot size of $\pi w_0^2 = 3.15 \times 10^{-8}\text{cm}^2$, we obtain for the energy

$$E = I_{\text{peak}} \cdot \tau = \Phi_{\text{th}} \cdot \pi w_0^2 \approx 60\ \text{nJ} \quad .$$

1) Docchio, F., Regondi, P., Capon, M.R., and Mellerio J. (1988) Study of the temporal and spatial dynamic of plasmas induced in liquids by nanosecond Nd:YAG laser pulses. 1: Analysis of the plasma starting times. *Appl. Opt.*, **27**, 3661–3668.